General Equilibrium Models in the Economy and in the Field of Development Policy

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Abstract

Economics is heavily reliant on mathematics, and with the advent of computers we can now make better decisions based on vast amounts of data. This article provides an overview of the mathematical structure underlying decision-making, which is motivated by elements of optimisation theory, which are currently supported by algorithms developed in equilibrium models in economics. We show a motivational overview of the theory of general equilibrium models and their advantage today. Although the approach of a computable general equilibrium model is a complicated task, it undoubtedly allows better decision making, especially in governmental decisions and little has been done in some developing countries.

Keywords: optimization, models, equilibrium, stochastic, development, simulations.

1. Introduction

1.1 Economic policy and decision-making

Global warming and an array of other environmental problems that are damaging the planet are examples of natural phenomena that concern science and are closely linked to human obligations. At the United Nations (UN) Millennium Summit in 2000[1], world leaders pledged to

[1]https://www.un.org/millenniumgoals/bkgd.shtml#:~:text=The%20Millennium%20Development%20Goals%20set,environmental%20sustainability%20%E2%80%94%20can%20be%20measured

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achieve the Millennium Development Goals (MDGs) by 2015, which relate to poverty, education, health, gender equality, the environment, and the creation of a global partnership for development.

This leads to the search for policies to find viable solutions and improve the quality of life on the agenda of developing country governments and the international community. Improving human conditions depends on the implementation of new policies that could have an impact on public budgets, especially in developing countries. In this context, and from an economic point of view, some qualitative and quantitative proposals have been made.

1.2 Computable general equilibrium models and economic development

One of the viable quantitative proposals has been the use of computable general equilibrium (CGE) models Castellanos, K., Feltenstein, A., & Sedrakyan, G. (2023), Burfisher (2021), as they make it possible to simulate different economic scenarios, which greatly assist in decision making.

CGEs have evolved from the first recognised work of Johansen (1960), where he explored aspects related to long-term economic development. Nobel Laureate Paul Samuelson states that one of the most controversial aspects of government policy is distribution, and that the dilemma is mainly between equity and efficiency. That is, ¿how much and how can resources be distributed without a loss of competitiveness? Samuelson himself (2005) argues that if economic growth is efficient and the income it generates is well distributed, it can be more beneficial to citizens; that is, more food and more housing, more resources for health care and pollution control, universal education for children and public pensions for retirees, among other development indicators. We agree with Samuelson that the fundamental role of the State is to promote social justice and economic balance, with governments as the main actors, whose minimum function is to be guarantors of a fair redistribution of the resources generated by constant and continuous economic growth.

There are several definitions of development; we understand development as those meanings that relate it in an eminent way to general well-being and the raising of living standards. The concept of development, according to Capalbo (2008), is a broader one that, although it includes progressive segments, allows for instances that, from a linear and quantitative point of view, can be considered as setbacks, but can also be highly significant, as turning points from which, an unforeseen or different course can be resumed, generating new hypotheses.

Bustelo (1999) mentions that since the 1940s, two main categories of development have been established, the orthodox theory of development, which is part of the mainstream of economics, and the heterodox or radical theory, which is characterised by its criticism of the former and by being resolutely outside the conventional approach; in which he points out that underdevelopment is simply a chronological problem, in which the least developed countries are in a state of backwardness in the modernisation or growth of their economic structures. On the other hand, the heterodox theory of development proposes that backwardness is more than a chronological problem, but a consequence of the disadvantageous position of poor or peripheral countries in the structure of the world capitalist system.

For Thirlwall (1987), the split represented by the Second World War challenged the hegemony of the neoclassical school and gave rise to the proposal of Keynes' theoretical thought. The Keynesian challenged the idea that a market economy would automatically lead to full employment. In this way, the loss of faith in the regulatory automatisms of the economy opened the door to the need for state intervention to achieve a situation of full employment.

According to Katz (2008), from the 1960s onwards, and after the contribution of the Solow-Swan growth model in 1956, economics abandoned the ideas of classical origin - which had come down to us in the writings of Young (1928), Hirschman (1958), Rodan (1943), Leibenstein (1957), Scitovsky (1954), Nurse (1957) and Myrdal (1957) - which gave a central role in the analysis of development to increasing returns to scale, Leibenstein (1957), Scitovsky (1954), Nurkse (1957) and Myrdal (1957) - which gave increasing returns to scale and externalities a central role in the analysis of development, and turned massively to the construction of what is now known as the modern theory of growth. This theory is expressed in terms of an equilibrium algorithm, in which development is equated with growth in output per capita and is studied within the framework of formal models in which institutions and uncertainty are absent, markets behave perfectly, economic agents are fully informed about the future, firms know consumers' tastes and have perfect access to the production functions they must use to satisfy them. On this basis, they maximise profits from a given set of exogenous data.

Katz (2008) argues that the development of an economy involves not only an increase in per capita output, but also structural changes in the economy, such as the creation of institutions, the building of markets and new technological and productive capacities, and the gradual formation of a country- and place-specific network of links and habits of interaction between firms. Consumers, government agencies and a wide range of other organisations - many of which do not necessarily operate according to market rules - such as universities, trade unions, local authorities, professional associations, and others.

On the political-ideological side, the rise of neo-liberalism in the 1980s, derived from neoclassical economic theory and strongly led by Washington and London, imposed a new international economic order based on the principles of the free market and economic growth through its international agencies such as the World Bank, the International Monetary Fund, and the World Trade Organisation. The approaches of economic neoliberalism assume a direct relationship between economic growth and national income, in which national income would trickle down to all members of society participating in the economic system, mechanically generating development.

Supported by the revival of neoclassical thought in the 1980s, the so-called neoclassical counter-revolution (1980-1990) was led by authors such as Bela Balassa, Anne Krueger, and Ian Malcolm David Little, who strengthened neoliberalism by criticising the basic needs approach, state intervention and the strategies of industrialisation through import substitution promoted by heterodox theories of capitalist development. These authors also passionately advocated internal liberalisation (reduction of the weight of the state) and external liberalisation (trade and financial liberalisation) in Third World countries. On the other hand, the basic needs approach emerged -

led by Paul Streeten, Mahbubul-Haq, Amartya Sen, Hans Singer, Richard Jolly, and others - which was supported by the United Nations Development Programme (UNDP) and later became the intellectual core of the Human Development Reports.

For several decades, some additional research has been carried out with different approaches, as shown in Devarajan and Robinson (2005), where analyses related to income distribution, international trade, poverty, etc. are found, but in developing countries, there is not much research related to computable general equilibrium models.

At present, the flexibility in the development of CGE models is great, since it allows them to be made by regions Partridge (2010) or multi-regions Giesecke (2002), Kim E. and Kim K. (2002), allowing a change in relation to the issues analysed in the beginning, especially due to the political decisions that arise in a civilisation as dynamic as the one we live in, where the flow of information is an important part of decision-making. Among the main influences that the models have had are those related to sectoral policies, energy, international trade, taxes, and global warming, although the interest is maintained in different areas of science. In this paper, we will have a look at the basic theoretical of general equilibrium models, as well as their development.

2. Theory and methods of research

2.1 Equilibrium models

Formally, from the point of view of economic theory, a general equilibrium exists when we have a mathematical expression that tells us that supply is equal to demand in an economy whose main activity is economic exchange. Its main characteristics have been discovered since its formulation by Jevons (1871) and Walras (1874), where the identity is rescued, called Walras' law.

To illustrate the structure of a general equilibrium model, let us consider the Arrow-Debreu model[2]. There are l goods, and where the space of consumption is a strictly positive *octant:*[3]

$X = \Re^l_{++} = \{x_i = \left(x_i^k\right) \in \Re^l / x_i^k > 0\}$ with $k = 1, \dots, l$ (1)

Let's consider the space:

$S = \{p = (p_j) \in \Re^l_{++} / p_l = 1\}$ (2)

where p is the normalized price. Let us m denote the number of consumers whose utility is represented by:

$u_i = X \to \Re$ (3)

[2]https://www.econometricsociety.org/uploads/Obituaries%20Past%20Presidents/arrow_geanakoplos.pdf March 6, 2024.

[3]An octant is each of the eight divisions of three-dimensional Euclidean space, defined by coordinate signs.

that satisfies a set of constraints in such a way that the demand:

$f_i : S \text{ x } R_{++} \rightarrow \Re^l_{++}$ (4)

and where $f_i(p, w_i)$ of consumers maximizes the utility $u_i(x_i)$ under the constraint $p \cdot x_i \leq w_i p$, satisfying $p \cdot f_i(p, w_i) = w_i$.

The parametric space $\Omega = X^m$ has the fundamental assumption that the economy is parameterized with the vector $w = (w_i) \in X^m$ and different parametric spaces can be selected depending on the interest in the research being conducted Arne Ryde (2006).

For this parametric space Ω, the equilibrium condition is generated under the assumption that $p \in S$ for a given price vector, the i-th consumer demand is given by $f_i(p, p \cdot w_i)$ in such a way that the excess of aggregate demand would be $z(p, w) = \sum_i f_i(p, p \cdot w_i) - \sum_i w_i$, which satisfies $p \cdot z(p, w) = 0$, an identity known as Walras' law of aggregate excess demand. Under these characteristics we will have that given an economy $w = (w_i) \in \Omega$, a price vector equilibrium $p \in S$ will be expressed as $z(p, w) = \sum_i (f_i(p, p \cdot w_i) - \sum_i w_i) = 0$ corresponding to the equilibrium condition, where the equilibrium set is denoted as $W(w)$, which is non-empty and can contain more than one element in such a way that the relation between $w \in \Omega$ and $W(w)$ is known as the Walras correspondence.

The solution to this type of problem was widely analysed, nearly 80 years ago; important works include those of Walras and Pareto (1909), as well as the contributions of Jevons, Edgeworth (1881), Hicks (1939) and Samuelson (1952). The main contribution was the solution to equilibrium conditions of the type $z(p, w) = 0$ with $w \in \Omega$, and the Pareto[4] optimization for corresponding equilibrium of $x = (f_i(p, p \cdot w_i))$.

Although the economic theory seemed to be a little fictitious with the reality, it was the work of Divisia (1928) where this type of problem is given proximity through the similarity with the elements of rational mechanics, giving birth to rational economics.

The fundamental step from the study of rational economics to modern general equilibrium economics is made with the study of the existence problem, addressed by von Neumann (1937).

The problems of existence and uniqueness were analysed by Wald (1936) in the theoretical development of general equilibrium, under some interesting assumptions. Faced with the problem of carefully establishing the assumptions for any rigorous solution of an existence problem, in general equilibrium models, the axiomatic approach emerged in 1940, supported by the contributions of great personalities of modern mathematics, including Cantor, Hilbert, Peano, Weierstrass (see details in Boyer, 2011). With this new vision, important results were obtained, such as the existence and welfare theorems, which led to intensive research in the 1950s and

[4] Op. cit.

1960s on the problems of singularity and stability of the solutions of the equilibrium equation, with Arrow, Block, Hahn, among others, generalising Wald's work on the singularity[5].

The theoretical work continued, and it is important to highlight the efforts made to mathematically formalise the theory supporting the general equilibrium models, but there are some doubts about the possibilities of finding general conditions on preferences or utility functions that would be compatible or imply the singularity and stability of the equilibrium, examples developed by Scarf (1960) and Gale (1963). However, the fundamental problem was solved by the concept of continuity and by Walras' correspondence:

$$W: \Omega \rightarrow S \qquad (5)$$

which led to the concept of regular economies. The solution given by Debreu (1970) was a landmark paper in which he established differentiability as the main tool of equilibrium analysis and showed that there exists an open subset R of W which has a complete measure, i.e., it is strictly necessary that its complement Ω/R has a null Lebesgue measure[6]. The approach that general equilibrium theory has followed is very important because its main goal is to establish properties that are true for all economies that can satisfy very reasonable and reality-bound assumptions.

Against this background, formal studies of general equilibrium problems in aspects of economic theory lead us to work in topological areas, since the equilibrium manifold leads to the related concept of natural projection, a map of the equilibrium manifold in the space of economies, and then to show that the natural projection is smooth and adequate, which is sufficient for this mapping to define a structure known in mathematics as a branching cover, so that Debreu's results[7] are a direct consequence of this structure. For a more rigorous analysis of the properties and conditions that must be satisfied, and to know the overall structure of a variety in equilibrium, some theoretical references can be consulted in Starr (1997).

Research on the mathematical foundations attached to economic theories marked a very important stage in their development, but Scarf (1969) changed the way of approaching the theoretical solution into a more attractive one, without losing the mathematical formality, by presenting an efficient numerical algorithm for equilibrium in complex economies, with the aim of finding real and practical solutions in aspects of public policy.

With the computational support introduced at that time and its application to different areas of public policy, it became popular and was called a computable general equilibrium (CGE) model. This type of model is generally used to quantify the impact of a change in trade policy on the welfare of countries and the distribution of income between countries or regions within several countries. By using computational methods, it is possible to analyse multiple policy changes and/or policy changes with ambiguous effects, where theory is not so easy to apply; in other

[5] Op. cit.

[6] Debreu named an element of R as a regular economy.

[7] Op. cit.

words, we can affirm that CGE models, by means of simulations, combine the abstract general equilibrium structure with economic data attached to reality to numerically resolve the levels of supply, demand and price that support the equilibrium in a specific set of markets.

The notion of an applied general equilibrium model was published by Shoven and Whalley (1992), where they state that the main basis is the construction of a Walrasian general equilibrium structure. With such a structure, the theoretical foundations are used to try to create a realistic model that best approximates the current situation. Real data, such as those recorded in the national accounts, are incorporated into these models in order to obtain the best approximation to reality.

The advantage of general equilibrium models is that the parameters can be varied, allowing several economic scenarios to be observed and compared in order to make better decisions.

3. Discussion

The fundamental idea of a system in equilibrium and its translation into a mathematical language was one of the best advances made by the economist Walras. In particular, he raised the need to study the necessary conditions for equilibria to be unique and stable, laying the foundations for what is studied today, but without doubt mathematical formalism, in the different turns that economic theory has presented, has been of great importance since it has allowed to quantify the dynamics presented by the different agents that make up economic development, allowing us to improve the approach to reality, especially with the support of computation and without ceasing to study the qualitative part.

The computational aspect is what has allowed great advances in the application of computable general equilibrium models, since most of the models are non-linear and require precision, and to be implemented in very specialized software such as GAMS[8], R[9], Python and other options available[10], which are the most popular in scientific research on optimization Heer and Maussner (2009).

It is important to note that today's computable general equilibrium models have evolved into dynamic models, with stochastic processes, complicated functional forms or parameter estimation using econometric methods, among others, in order to best reflect economic reality.

However, it is necessary to adapt these results to the technical areas generated in government bodies, so that their interpretation has the desired impact at the time of decision-making in the field of public policies, in order to ensure that balanced economic growth leads to economic development for the region or country where the model is applied.

8 https://www.gams.com/

9 https://cran.r-project.org/web/packages/GE/GE.pdf
https://gecon.r-forge.r-project.org/

10 https://www.uni-augsburg.de/de/fakultaet/wiwi/prof/vwl/heer/dgebook/
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