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**PRICING STRATEGIES OF PRIVATELY-OWNED SOCIAL HOUSING PROJECTS : A CASE STUDY OF TAICHUNG CITY, TAIWAN**

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**Abstract**

In Taiwan, the term “public housing” is also called “social housing” and the residential market is facing problems of high housing prices, high vacancy rates, and insufficient transparency in the leasing market. Since 2011, the government has promoted a policy of renting and not selling social housing, and through related plans, it has assisted socio-economically disadvantaged households to rent housing.

Real options theory (ROT) originated in 1977, Real option valuation model (ROV) has been utilized in a variety of real estate development decision, from planning to operations and from operations to abandonment.

This study takes the four social housing cases of A, B, C and D (three of which have been constructed) currently open for rent in Taichung City as examples, uses the net present value (NPV) and ROV methods to evaluate the benefits of investing in real estate. Through the embedded real options adopted at decision making stage and the Black-Scholes model modified with value leakage, this study using the geometric Brownian motion to calculate the corresponding option premium (OP). Adding OP and passive NPV together, the expanded net present value (ENPV) is obtained as the investment value of a privately-owned social housing project. Both the development model and valuation model proposed in this study have been verified in the aforementioned four social housings. The net present value of these four places is negative, and the ENPV of three of them will be positive after a one-year deferral of development. Finally, through sensitivity analysis, it is found that the average rent increase rate of social housing is the most influential indicator affecting the fluctuation of the ENPV. Taking case A as an example, when the average rent drops by less than 3%, the ENPV will become negative. The second largest influencing indicator is the interest rate of loan funds. Taking case A as an example, when it increases by about 5%, the ENPV will become negative.

To put the idea of privately-owned social housing projects into practice, a project financing mode titled BOO (Build-Own-Operate) is employed in this study, and the proposed valuation model can objectively and comprehensively value a privately-owned social housing projects.

**Keywords:** Public housing, Net present value, Real option valuation model, Sensitivity analysis, Geometric Brownian motion

## **1. Introduction**

In Taiwan, the residential market is facing problems of high housing prices, high vacancy rates, and insufficient transparency in the leasing market. Since 2011, the government has promoted a policy of renting and not selling social housing, and through related plans, it has assisted socio-economically disadvantaged households to rent housing (CPAMI, Construction and Planning Agency, Ministry of the Interior, R.O.C., 2018c).

The term “social housing” is a term introduced by European countries. The history of social housing in Europe began more than a century ago. Following the brutal “social warfare” that marked the first stages of industrialisation, philanthropic and industrial interests began tentatively to develop social dwellings. The sector grew between the two world wars and then more strongly after World War II. Its stakeholders (banks, local and national governments, employers, unions, architects and planners) variously envisioned social housing as a key component of welfare policies and/or an important tool in the political local power balance (Scanlon et al., 2014).

The term “social housing” is also called “Social Rented Housing” in Europe (more emphasis on its spirit of “rent only, not for sale”). It is also called “Public Housing” in the United States, also called “Public Rental Housing”, “Government-Owned Housing”, “Public Housing”, “Collective Housing”, “UR housing”, etc. in Japan, called “Public Rental Housing” in Hong Kong, also called “Housing and Development Board” in Singapore, and also called “Public Rental Housing” in China. As to the term “social housing” (called Sociale huisvesting in Dutch and logement social in French) in Netherlands, United Kingdom, and France is the same as in Taiwan.

Since Taiwan was under Japanese colonial rule, the “Public Housing Plan” has been the main residential policy in Taiwan. The government centralize the construction and management of the public housing constructed for sale or rent to low-income families, or conducted by such families themselves with government loan, or provides the subsidy of loan interest for families who bought the house to solve housing problems and enhance the quality of life. Thereafter, the government provided different housing subsidies for different occupations, including the housing plan of the in-service military personnel, the housing plan of civil servants, policemen, elected representatives, public school teachers, the housing plan of workers, farmers, fishermen, and the housing plan of indigenous peoples, low-income households, people with disabilities, elders, women at high-risk, etc. (Institute for Physical Planning & Information, 2012). Refer to Articles 3 and 4 of the “Housing Act” promulgated on January 11, 2017 in Taiwan, the term “social housing” refers to housing and necessary facilities built by the government or by the private sector with subsidies from the government that is primarily rented. At least 30% of social housing built by the competent authority and private sector calculated based on the jurisdiction of each municipal, county (city) government shall be rented to economically or socially disadvantaged persons. A specific ratio of social housing shall be provided to persons whose permanent address is not registered in the area where they are going to school or working in.

According to the "Social Housing development or construction plans" approved on March 6, 2017, in Taiwan, the social housing policy goal (building 200,000 social housing in eight years)

was mainly achieved through two methods. Firstly, the government build social housing directly, and secondly, the government rent and manage a private building through "House Leasing and Rental Management" policy. The medium-term goal of this policy is set from 2017 to 2020, and to complete the supply of 80,000 social housing, including the government build 40,000 households directly and provide 40,000 households through "House Leasing and Rental Management" policy. The total supply of 200,000 households of social housing is expected to be reached by 2024, including the government build 120,000 households directly and provide 80,000 households through "House Leasing and Rental Management" policy. (CPAMI, 2018c).

Refer to Articles 19 to 21 of the "Housing Act", social housing may provided by the competent authorities or private organizations. The competent authorities may provide social housing via the following methods: (i) Construction. (including direct building, Co-construction, Set up superficies rights for private participation in building, Participate in urban renewal with public land or building and gain a building and its site, and other method determined by the competent authority.) (ii) Use a public building and its site. (iii) Accept a donation. (iv) Purchase a building. (v) Rent and manage a private building for sublease. (vi) Provide incentives, guidance, or subsidies for house rental service providers specified in Paragraph 2 of Article 52 of Housing Act to rent and manage private buildings for sublease, or provide matchmaking and management services for landlords and tenants. (vii) Change registration of land use and donations for bulk reward. (viii) Other methods determined by the central competent authority.

The second item mentioned above is to make good use of a public idle building and its site to transform into social housing, which is a very effective way to provide social housing, but it needs to undergo seismic assessment or structural reinforcement to ensure the safety of the building structure. Therefore, use a public building and its site to transform into social housing may be classified as a construction and maintenance project, or classified as the aforementioned construction of direct building. Refer to the relevant interpretation of the Ministry of the Interior, R.O.C., the competent authority can still provide social housing through two methods. Firstly, provide multiple social housing through urban planning, and secondly, through "The Operating Procedures of Participate in Urban Renewal or Joint Venture with Public Land of Government Agencies and Gain a Building and Its Site which would be Preferentially Used as Public Rental Housing or Social Housing" policy. In addition, Private organizations may provide social housing via the following methods: (i) Construction. (ii) Addition, reconstruction, and renovation of building on the same site. (iii) Purchase building. (iv) Rent and manage private building for sublease. (v) Other method determined by the competent authority. Therefore, it can be used multiple methods and channels shown in Fig. 1. in Taiwan.

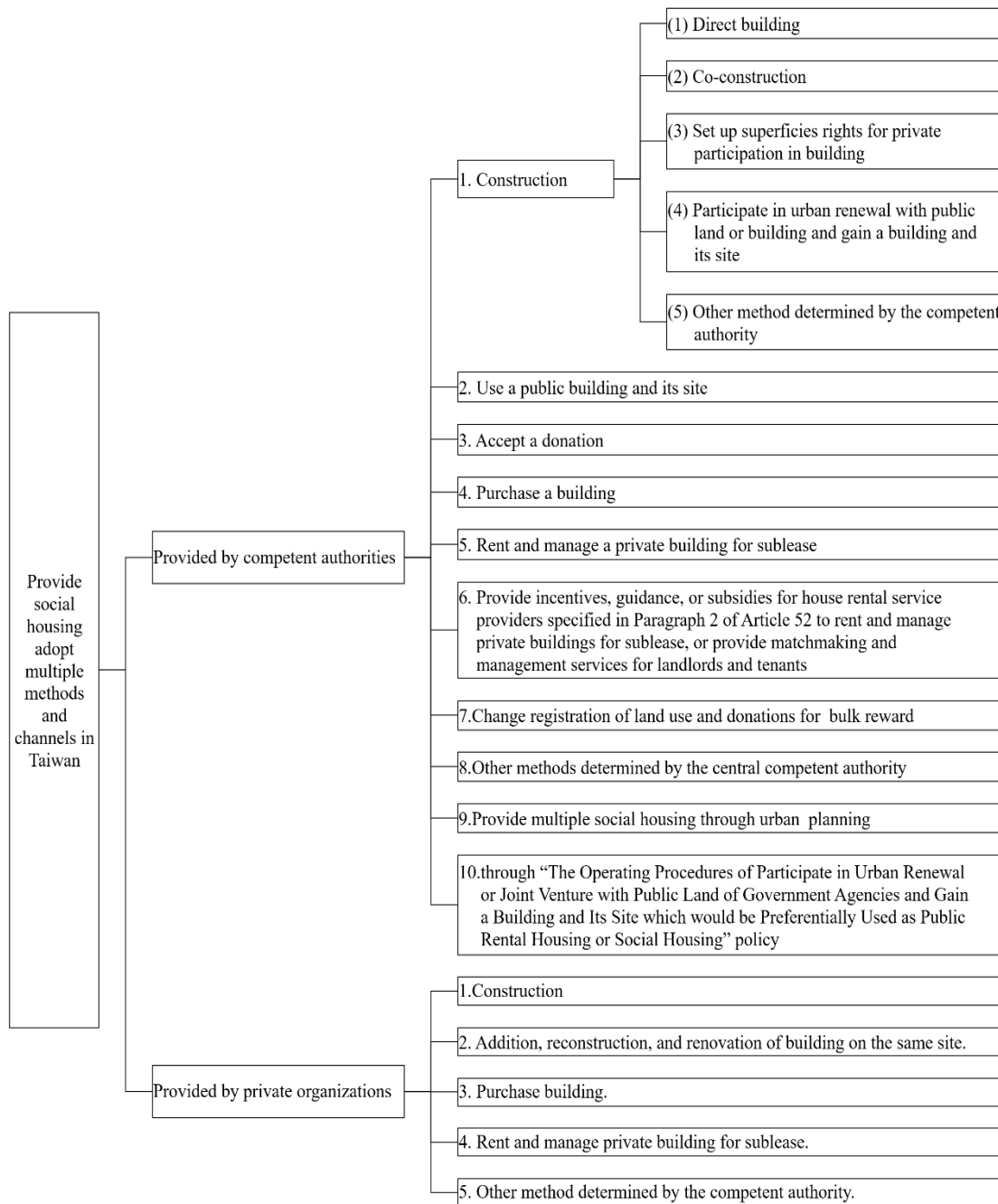


Fig. 1. Provide social housing adopt multiple methods and channels in Taiwan

Refer to Articles 3 of the “Housing Act”, The initiating entity of social housing can be divided into the government or the private sector. At present, to assist local government in building social housing, central government currently has provided many ways of assistance, such as subsidies of preliminary plan fee, long-term lease of public land, the establishment of a financing service platform, financing interest during construction and non-self-reimbursable cost subsidies

during operation, and full subsidies of the promotion fee of “House Leasing and Rental Management” policy (CPAMI, 2018a).

In addition, when it comes to the competent authority to use the first method to construct social housing (direct building), the budget required is the largest. The pure rental income may not be able to afford the operation, maintenance and management expenses, and the financial plan should be balanced through budget allocation and other supporting programs. Based on the limited financial resources of the government, the construction and management of public construction through the private sector, and the introduction of corporate management concepts to improve the quality of public services (such as the aforementioned second to fifth methods of the competent authority for constructing to provide social housing), has become an international trend. In the broad sense, the private sector includes not only institutions of private enterprises, but also common non-governmental organizations (NGOs) or non-profit organizations (NPOs), which usually belong to the third sector (Taiwan Competitiveness Forum, 2015).

The provision of social housing in Taichung City can be divided into two parts: construction and through “House Leasing and Rental Management” policy. In the part of construction, it is currently divided into two stages. The first stage aims at 5,000 households in four years. The current volume of construction is 5,008 households, which will be completed in 2022 and divided into sixteen cases, of which fourteen are government self-built, one is a case of donation (divided back) for volume awards, one is a case of multiple construction through urban planning means (feedback), and of which three cases (a total of 591 households) have been completed (Urban Development Bureau of Taichung City Government, 2020). However, of the 14 government self-built cases, there is only one of the fourteen cases planned to be invested by institutions of private enterprises (using the BOT method), which means that few institutions of private enterprises are active in providing social housing. In related research, very few of China’s private sector are found to be active in the provision of public rental housing, while the main reason is the disability of conventional net present value (NPV) method in dealing with great uncertainty in public rental housing project (Zhong, 2013).

Therefore, based on the limited financial resources of the government, and to effectively attract institutions of private enterprises to provide social housing, a privately-owned provision mode of social housing projects will be created in this study, where the private sector has full ownership and flexible decision rights of the projects, complying with all relative regulations and enjoying all incentive policies related to social housing simultaneously. Then, utilizing real options theory (ROT), real options of this new model at decision making stage will be identified by institutions of private enterprises. In addition, a real option valuation model (ROV) for this new model will be proposed, followed by a case study of the four social housing cases (three of which have been completed for construction) currently open for rent in Taichung City to conduct relevant research.

## **2. Literature review**

The Netherlands is currently the oldest country in the world to develop a social housing system and has more than 150 years of practical experience. It is also the country with the highest social

housing stock ratio in the world. The new Housing Act (called *Nieuwe Woningwet* in Dutch) of 2015 revised the role of the country's housing corporations (HCs). In 2017, the total population of the Netherlands was about 17.08 million, of which social housing stock was about 2.28 million units, accounting for about 30% of the total housing stock (Pittini et al., 2019). The UK government outlined its ambitions for the social housing sector in a 'Green Paper' in August 2018. In 2017, the total population of the Netherlands was about 66.04 million, of which social housing stock was about 5 million units, accounting for about 17% of the total housing stock (Pittini et al., 2019). In France's case, in 1912, the "Bonneville Act (called *la loi Bonnevey* in French)" was passed, which authorized local governments to establish public housing institutions for low-cost housing, such as the "Paris Habitat" established in 1914 as a special unit for the construction and management of social housing (Tsai, 2020). After 100 years, in 2017, the total population of France was about 66.77 million, of which social housing stock was about 5.2 million units, accounting for about 16.5% of the total housing stock (Pittini et al., 2019). In most European countries, a strong public housing sector provides affordable and good quality housing to a substantial share of the population. From a public policy viewpoint, understanding how public housing shapes the labor prospect of individuals, as well as their later housing consumption, is essential (Goffette-Nagot & Sidibé, 2016). The difference between non-profit organizations and profit organizations is essential in markets with rent control and that non-profit organizations may reduce some of the market distortions induced by rent control (Van Ommeren & Koopman, 2011).

In 2010, the total population of Japan was about 128.55 million, of which public rental housing (common terms for social housing in Japan refer to government-owned housing, public housing, UR housing, etc., hereinafter all referred to as public rental housing) stock was about 3 million units, accounting for about 6.06% of the total housing stock (Institute for Physical Planning & Information, 2012). However, in Taiwan, as of February 2019, the total population was approximately 23.6 million, of which social housing stock was approximately 15,912 units, (CPAMI, 2020), accounting for about 0.18% of the total housing stock.

Due to various objectives like obtaining the private sector's finance and expertise, allocating risks, improving service quality and lowering life-cycle cost (Abdul-Aziz & Jahn Kassim, 2011), the private sector has been adopted broadly to promote the provision of social housing. For example, there are currently about 378 social housing associations in the Netherlands, which manage 2.4 million social housing units. They own social housing ownership, and are responsible for construction, operation and management. The lands acquisition of those associations come from the government's cheap rental land and are subject to considerable control. By providing policy incentives and the establishment of residential legal entities by the third sector, the social housing is provided by the specialized agencies of residential legal entities (Taiwan Competitiveness Forum, 2015). That is, the government entrusts a private NGO or NPO, Dutch Housing Association, a non-profit third-party independent organization formed as a cooperative to develop, build, and manage (CPAMI, 2018b). The construction of public housing in Japan has played a great role in solving the problem of housing shortage in Japan. Under the leadership of the Japanese government, a public housing supply system with housing provided by local governments, UR housing and housing constructed with a low-interest loan provided by

the Government Housing Loan Corporation as the three pillars has been formed. According to the “Act on Public Housing”, the first pillar, housing provided by local governments, is a low-standard, low-rent public housing constructed and leased by local governments. The second pillar, UR housing, is a public housing provided by the “Residential Public Corporations” (now called “urban regeneration agency”) established in 1955. UR housing is mainly rented to middle-income families, including leased housing and saleable housing. The last pillar is the housing constructed with a low-interest loan provided by the Government Housing Loan Corporation. (Taiwan Competitiveness Forum, 2015). These residences are managed by the central government, residential public corporations or local governments (CPAMI, 2018b). In the 1990s, Japanese public housing decided delegate initiatives for the provision of that housing to the private market. In order to supplement that, in the Housing Safety Net Law of 2007 it clearly affirms that the central government must pay heed to the appropriate apportionment of low rent housing that it is possible for low income earners to live in. And it aims at improving the functioning of the housing safety net through the effective utilization of public rental housing stock and the fostering of public housing by leasing existing private rental housing (Toshio, 2011). The stock of social housing in the United States accounts for about 6.2% of its housing stock. It was built under the government's lead in the early post-war period. In the 1970s, due to race, discrimination and other issues, it was built through massive subsidies for private entry. It is characterized by the US federal government through various active financial mechanisms, so that private construction companies or non-profit organizations are also willing to participate in the construction of social housing (Organization of Urban Re-s, R.O.C., 2015). American public housing has similar experience in its 75 years history, which gradually shifted from sole public efforts to promoting mixed finance partnerships between the public, private, and non-profit sectors (Vale & Freemark, 2012).

According to the development and acquisition of social housing, the social housing currently under implementation in Taiwan is divided into the following three modes of provision, namely the first sector (Public Sector, government) provide mode, the first sector and second sector (Private Sector, private institutions, usually builders), public sector and private sector, provide together, or the third sector (private legal persons, private third sector, refers to government subsidies for private non-profit organization and social enterprise established to promote the social housing policy internationally) provide mode (such as the social housing of Gospel and Welfare Service Community in Dalin, Tainan city, established by the Eden Social Welfare Foundation in Taiwan, which has this property). In addition, the "Housing Act" and "Act for Promotion of Private Participation in Infrastructure Projects ” are the current legal sources for the Taiwanese government to encourage private participation in the provide of social housing. According to the “Housing Act”, the private organizations are able to provide social housing by addition, reconstruction, and renovation of building on the same site; according to the “Act for Promotion of Private Participation in Infrastructure Projects ”, the private institution is able to provide social housing via the methods of Build-Operate-Transfer (BOT), Build-Transfer-Operate (BTO), make payments for Build-Transfer-Operate (Payment-BTO), Rehabilitate-operate-transfer (ROT), Operate-Transfer (OT), Build-Own-Operate (BOO) or any other way approved by the competent authority (Taiwan Competitiveness Forum, 2015). Social housing is a

public property. For the government and the general public, it can create good external benefits and social benefits, but it has the characteristics of low self-compensation ability and has become a restriction of private participation (Huang, 2019). Internationally, Public-Private Partnership (PPP) is considered to be a form of structured cooperation between public and private parties in the planning, construction and/or exploitation of infrastructural facilities in which they share or reallocate risks, costs, benefits, resources and responsibilities (Koppenjan, 2005). Institutionalized PPPs (iPPPs) are hybrid organizations (e.g., alliances, joint ventures, and mixed companies) in which public and private partners jointly manage and deliver services (Hodge, Greve, & Boardman, 2011 ; Reissner, 2017). Contractual PPPs (cPPPs) entail a transactional relationship based on designing, monitoring, and enforcing contracts in which public-sector outsourced IPSs revert to private partner (DA CRUZ & MARQUES, 2012). Regulated PPPs (rPPPs), also known as public-private collaboration, are collaborative relationships between the public and private sectors in which the private sector independently owns the assets and provides services under the public sector's regulations (Crispeels et al., 2018; Schaeffer & Loveridge, 2002). Through the PPP-BOT model, the government uses private and public sector cooperation to introduce private capital, technology, capabilities and efficiency to solve the government's financial shortcomings, improve the quality of public construction and improve the welfare of the whole society. To provide by the private sector can reduce the possibility of social housing being labeled, and by adopting the mode of releasing subsidiary businesses to private investors, such as restaurants, parking lots, stores and hotels, etc., can increase the self-liquidating ratio (Huang, 2019). The governance structure of cPPPs dominates China's PPP projects, including concessions and PFIs (Xiong et al., 2020). PFIs were officially adopted by China in 2014. The MFC applied a regulatory framework modeled on the British PFI to Category C projects, such as education, health, social housing, environment, lighting, equipment, information and communication technology, and entertainment facilities (HM Treasury, 2017). In the next few years, Housing PPP will still be adopted in Malaysia in years to come. Government agencies in Malaysia can take note of the success factors and failures factors so as to ensure that the lands they invest in housing PPP projects are not squandered. Their objectives of employing this approach should also be realistic; otherwise their expectations would not be met (Abdul-Aziz & Jahn Kassim, 2011).

A real option is the right, but not the obligation, to take an action (e.g., deferring, expanding, contracting, or abandoning) at a predetermined period of time – the life of the option (K, 2014). Like their financial cousins, the value of real options depends on five basic variables (although others may come into the picture), plus an important sixth. The six are: (i) The value of the underlying risky asset. (ii) The exercise price. (iii) The time to expiration of the option. (iv) The standard deviation of the value of the underlying risky asset. (v) The risk-free rate of interest over the life of the option. (vi) The dividends that may be paid out by the (Copeland, 2002). Real options theory originated in 1977 with the ground-breaking idea of Stewart Myers that Black-Scholes financial option pricing model developed in 1973 can be applied to capital-budgeting, later it was proved by Folta & O'Brien (2004) and Borison (2005). Myers (1977) originally defined "real options" as: "opportunities to purchase real assets on possibly favourable



terms”(Čirjevskis & Tatevosjans, 2015). From the research of Merton(1998), the future is uncertain (if it were not, there would be no need to create options because we know now what we will do later) and in an uncertain environment, having the flexibility to decide what to do after some of that uncertainty is resolved definitely has value. Option-pricing theory provides the means for assessing that value.

Refer to Li et al.(2014) and this study, Real option valuation model (ROV) has been utilized in a variety of real estate development decision, from planning to operations and from operations to abandonmen (Hui et al., 2010). Some studies apply ROV to predict land prices (Grovenstein et al., 2011; Shen & Pretorius, 2013) or rent of soil (Hsieh & Lin, 2016), and some scholars utilize ROV to value certain types of real estate development, such as recreational facilities (Leung & Hui, 2002), public housing upgrading (Ho et al., 2009), office construction (Fu & Jennen, 2009) and farm (Stokes, 2012).

From above literature review, it is clear that the second sector and the third sector have been worldwide adopted in the provision of social housing, and it has really done well in numerous circumstances (Li et al., 2014). These international experiences may provide reference for promoting the private sector's provision of social housing in Taiwan. At the same time, as an efficient valuation model, ROV has been broadly applied in decision making of different types of real estate development. However, the application of ROV in valuing social housing projects provided by private organizations (privately-owned social housing projects, mostly were construction by the competent authorities attracted the second sector to investment and construction) at decision making stage is rarely found so far. This study aims at filling the knowledge gap by constructing a valuation model for privately-owned social housing projects at decision making stage, based on the identification of imbedded options in such projects.

### **3. Scope and methodology**

#### **3.1 Characters of the privately-owned social housing projects with BOO fundraising model**

Because of special housing history and national conditions, non-profit housing organizations have been stifled in a long time and thus almost disappear in China's housing provision system (Wang, 2012) ° In contrast, China's private real estate developers have developed and grew in the booming commodity housing market since 1980s (Li et al., 2014). Generally speaking, the private organizations can be divided into institutions of private enterprises (profit-making private sector) and NGOs or NPOs (non-profit-making sector, usually belonging to the category of the third sector)(Taiwan Competitiveness Forum, 2015).

The research object of this study is the part of "private sector" participating in the provide of social housing. Besides the five ways provided by private organizations in Fig. 1, it also includes the two construct methods, co-construction & set up superficies rights for private participation in building, the seventh method (change registration of land use and donations for bulk reward) provided by the competent authorities, and the government entrusts the private organizations to operate as shown in Fig. 2. Among them, the two construct methods, co-construction & set up

superficies rights for private participation in building, are provided by the competent authorities attracted the second sector to investment and construction.

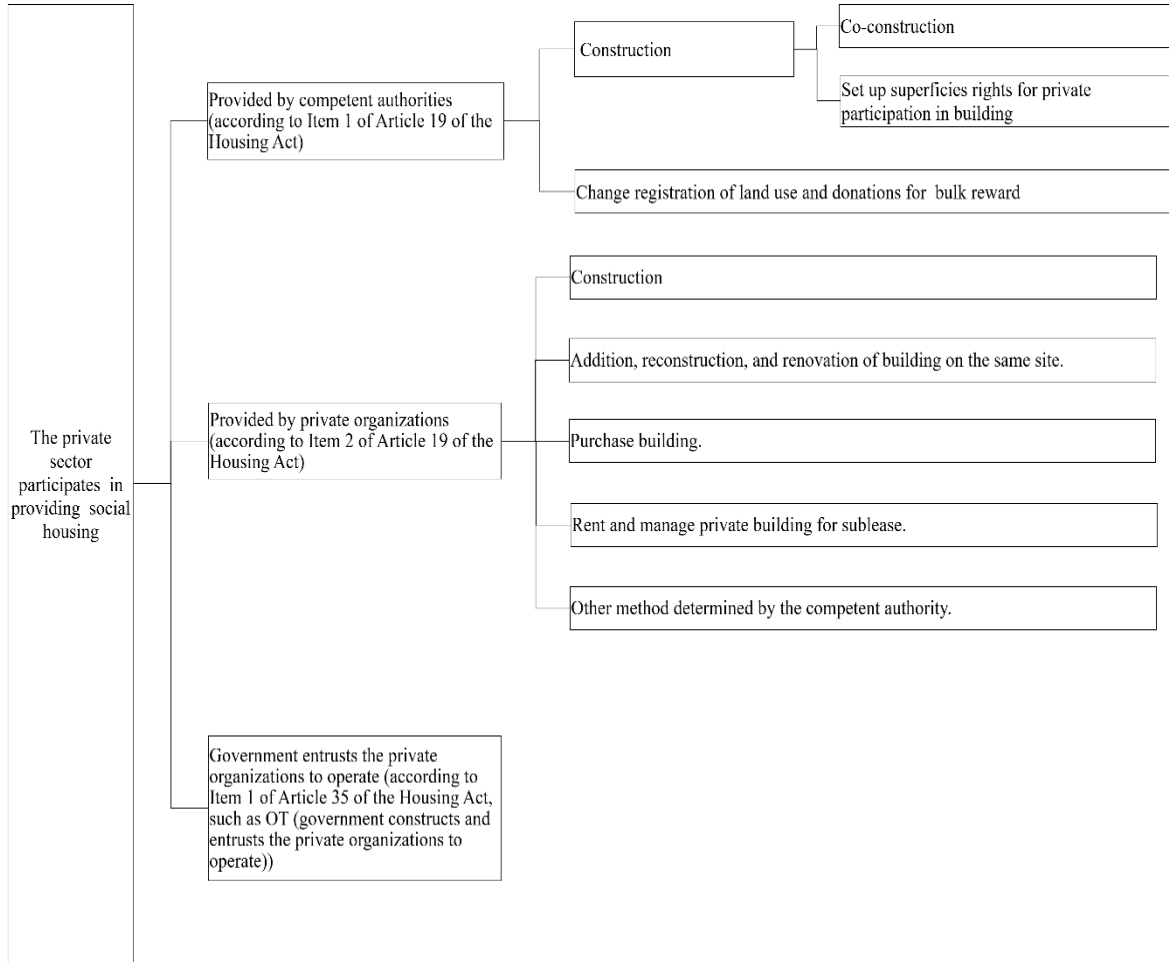


Fig. 2. Private sector participates in providing social housing in Taiwan

Therefore, the private sector participates in providing social housing can be divided into private-initiated or government-initiated, but currently in Taiwan, the cases of the private sector participates in providing social housing all belong to the latter. The main reason is that the existing real estate structure is mainly for sale of residential buildings with a certain profit. The number of professional rental residential houses in the general market is quite scarce, and social housing with the rent below the market rent will be more difficult to operate. So far, there is no case of private-initiated. So, the “private sector provide” in this study principally refers to initiated by the government. As for the tenants, if they qualify for social housing support, they should be economically or socially disadvantaged persons, or persons whose permanent address is not registered in the area where they are going to school or working in. Besides, their annual household income must be lower than the average income of 50% of households in the

municipality or county (city) where the social housing is located at the time of application, and they should be persons that do not own a house.

As shown in Fig. 2, in Taiwan, the ways private sector participates in providing social housing (direct building), including government-initiated, private-initiated, and the government entrusts the private organizations to operate (for example, OT). Through the power of private organizations to promote social housing, more people in need would be taken care of with housing resources (CPAMI, 2019).

To put the idea of privately-owned social housing projects into practice, with reference to Li et al., (2014); CPAMI (2018c), a project financing mode titled BOO (Build-Own-Operate) is employed, and the privately-owned social housing model is shown in Fig.3. At this time, the social housing is provided via construction by the private organizations. Based on the limited financial resources of the government, it has become an international trend to build and manage public construction through private investment, and to introduce corporate management concepts to improve the quality of public services (Taiwan Competitiveness Forum, 2015). In addition, according to Article 19 and Article 29 of the Housing Act, the way private sector participates in providing social housing (SH) can still adopt a project financing mode titled BOT.

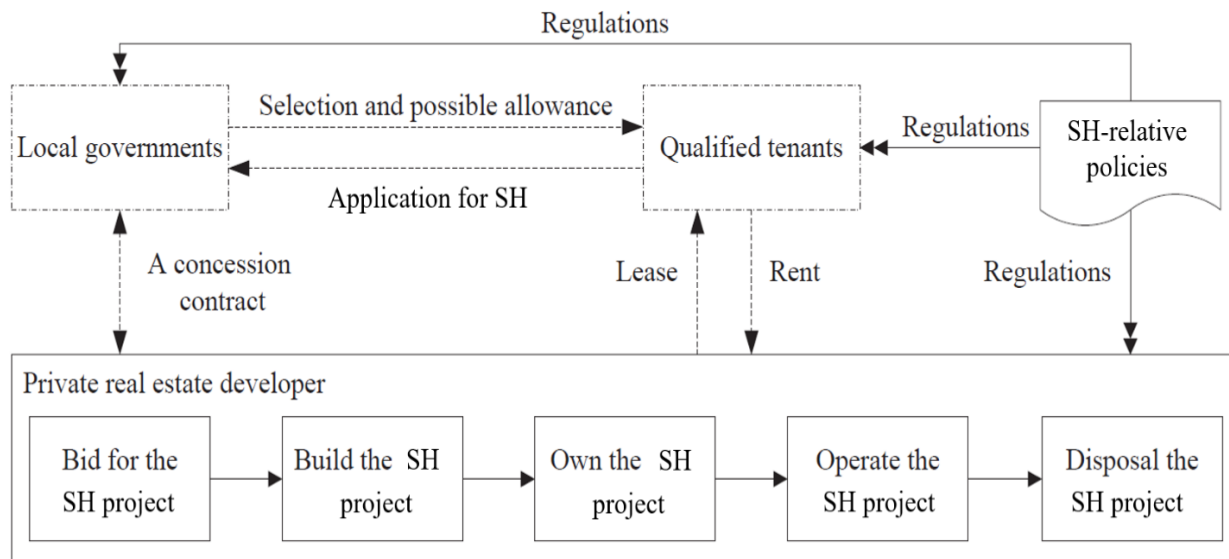


Fig. 3. The proposed model of privately-owned social housing projects.

In Taichung City, there are currently 5,008 social housing units provided, divided into 16 cases, all of which are provided by the competent authority, and there are no cases provided by the private sector. The purpose of social housing is to provide rental housing of a certain quality at a relatively low price, in other words, a subsidized housing policy. Therefore, if the mode of encouraging private sector to provide, how to balance the dilemma of implementing the effect of subsidies (principle) and the financial feasibility (technical) of private organizations has always been a challenge and the key. The government encourages the private sector to build social

housing, whether this “private sector” is a residential legal person, non-profit organization, social enterprise, construction company or life insurance company with foreign experience, sustainable finance is a necessary condition. The difference is that different sponsors have different levels of “reasonable or practicable” financial returns and management and operational risk allocation. Social housing may not be self-compensating for the purpose of achieving policy. Therefore, if we want to encourage the private sector to provide, we must provide positive incentives for financial subsidies.

Only through the financial subsidy assistance and incentive design of these different channels can we mobilize the flexibility of the private sector to participate in the construction of social housing under the policy objectives (increasing inventory, object protection, rental concessions, etc.) (Organization of Urban Re-s, R.O.C. , 2015).

In Fig. 3, it is evident that the private real estate developer is responsible for the life-cycle management of the social housing project, including its fund-raising, construction, possession, operation and final disposal. All those rights and obligations should be clearly stated in a formal concession contract between the private real estate developer and local governments, who also play several roles (Li et al., 2014), including “enabler” by providing the enabling environment for the private real estate developer to step in, “moderator” by balancing market incentives of the private real estate developer with public interests of the tenants (Sengupta, 2006), and “regulator” by establishing policies for the stakeholders including private real estate developer and qualified tenants (Leung & Hui, 2005).

In order to reward private sector to provide social housing, Taiwan and Taichung City have also issued relevant incentives for private sector to provide social housing in terms of financial subsidies and tax relief. According to Articles 2 and 3 of the “Regulations on Preferential Treatment for Rent and Setting up Superficies for Private Organizations to Provide Social Housing by Using Non-public Use Publicly Owned Property”, relevant financial subsidy measures have been stipulated for the royalties of superficies, the rent for rent and setting up superficies, and the rent of public buildings. According to Article 4 of the “Taichung City Self-Government Ordinance for Providing Preferential Treatment on Social Housing Provision and the Houses Leased out by Public Welfare Landlords”, the relevant tax reduction and exemption measures have been established for the basis and scope of the reduction and exemption of land value tax and house tax during the provision of social housing. Related incentives (shown in Table 1) need to be considered in the model proposed in this study. It is important to note that these tax preferential policies are only applicable to social housing buildings in the privately-owned social housing projects. That is to say, other affiliated businesses (such as commercial buildings) constructed to increase investment incentives of private sector cannot benefit from the above tax incentives.

Table 1. Incentives for rewarding private sector to build social housing

Item Number	Operating expenses or tax denomination	Basis	Ratio or tax rate	Deducted ratio
1	Operating expenses			
1-1	Royalties of superficies	Assessed present value	10%	0%
1-2	Rent for rent and setting up superficies			
1-2-1	From the effective date of the contract to the period before the approved operation	Land area & declared value	More than 10%	0%
1-2-2	During the operation period	Land area & Declared value & The ratio rented to economically or socially disadvantaged persons	More than 10%	Calculated according to the ratio rented to economically or socially disadvantaged persons
1-3	Rent of public buildings			
1-3-1	From the effective date of the contract to the period before the approved operation	Current value of the house	Less than 3.6%	0%
1-3-2	During the operation period	Current value of the house & The ratio rented to economically or socially disadvantaged persons	Less than 6.1%	Calculated according to the ratio rented to economically or socially disadvantaged persons
2	Tax items		Tax rate	
2-1	Land value tax	Land area & declared value	More than 10%	80%
2-2	House tax	Current value of the house	Less than 3.6%	20%

### 3.2 NPV at decision making stage of the privately-owned social housing projects

Traditionally, discounted cash flow (DCF) analysis is used to value projects, companies or assets, and the time value of money is calculated at the discount rate. The sum of all discounted cash flows each year is the net present value (NPV), can be calculated as:

$$NPV = \sum_{t=0}^n \frac{(CI_t - CO_t)}{(1+q)^t} \quad (1)$$

where t means time in unit of year; n means the life-span of the privately-owned social housing project;  $CI_t$  and  $CO_t$  means the cash inflow and cash outflow of the privately-owned social housing project at year t separately; q means the discount rate.

From Equation (1), it is evident that three elements are crucial to estimate the NPV of a privately-owned social housing project, namely  $CI_t$ ,  $CO_t$  and q. As for  $CI_t$ , its principal source is the rent income of social housing buildings, including social housing rental income, parking spaces rental income and other (shops and social welfare facilities of residential services for people with disabilities, kindergarten, etc.) rental income, which equals to the product of its total building area, average rent, average letting rate and operation period (need to deduct the annual idle period). The total building area is generally determined by the land area and maximum floor area ratio, can be determined by the calculation results of the preliminary plan of the case. With regard to the average rent, the law assumes 70% of that of similar private rented housing buildings nearby. In most cases, a certain quantity of other affiliated businesses (such as commercial buildings) will be allocated in social housing projects as a subsidy to improve the financial performance. With regard to the average letting rate, it can be estimated from comparison analysis with running social housing projects in the same city, or from the demand and supply situation of local social housing. As for the operation period of social housing buildings and other affiliated businesses (such as commercial buildings), generally, it is estimated that the construction period is 3 years and the operation period is 50 years (Taiwan Institute of Property Management, 2018 ; Dragonpolis Engineering Consulting, 2019).

As for  $CO_t$ , there are four main sources of a privately-owned social housing project, namely the purchase cost of land, capital expenditures (construction cost of buildings), operating expenditures and loan interest. In this study, capital expenditures (construction cost of buildings) includes plan, design and construction supervision cost, direct construction cost, project administration fees and others (including project management cost, price adjustments, the cost of public art, various administrative fees, etc.)

As for q, with reference to Article 43 of the “Regulations on Real Estate Appraisal”, the discount rate should be determined from a comprehensive review of the methods, including risk premium method, market extraction method, weighted average capital cost method, debt coverage ratio method, and effective gross income multiplier method. Internationally, in addition to the aforementioned methods, q can still be calculated by several quantitative methods, including Capital Asset-Pricing Model (CAPM), Multifactor Asset-Pricing Model (MAPT) and Arbitrage Pricing Theory (APT), or qualitatively set by management as a requirement for the firm, or as hurdle rate for specific projects (Mun, 2005).

In terms of financial feasibility assessment, the financial feasibility and balance mechanism need to be considered at the same time. However, public welfare and investment profits are often not available at the same time. To increase the rate of return and reduce risks, the public sector must propose relevant subsidies and guarantee mechanisms, ensuring that private real estate developers have expected profits. In order to promote financial feasibility, it is necessary to

adopt a mode of releasing other affiliated businesses (such as commercial buildings) to private investors. The operation of the government to release other affiliated businesses (such as commercial buildings) must meet the following four conditions: (i) NPV is greater than zero ( $NPV > 0$ ). (ii) Internal rate of return is greater than the weighted average cost of capital ( $IRR > WACC$ ). (iii) Discounted payback period is less than concession period ( $DPB < Concession$  Period). (iv) Self-liquidating ratio is greater than 1 ( $SLR > 1$ ) (Organization of Urban Re-s, R.O.C., 2015; Huang, 2019).

### **3.3 ENPV at decision making stage of the privately-owned social housing projects**

Options are usually divided into European options and American options, and the sole difference between them is holders of one European option are only allowed to exercise the option on the maturity date, whereas one American option can be exercised at any time during the expiration period (Hui et al., 2010). Due to this additional flexibility the American options can be more valuable. In order to avoid arbitrage the price must be always at least the same as the available payoff. A put option gives the right to sell the underlying asset for a specified strike price while a call option gives the right to buy the asset for a strike price (Balajewicz & Toivanen, 2017). As regards real options which are applicable to real estate development, they may be summarized as a deferral option, an abandonment option, an option to expand/ contract, a switching option, a growth option and a compound option (Trigeorgis, 2005 : Guma, 2008). In the international literature on land development and public construction, there are three common options: a deferral option, an option to expand and an abandonment option (Chen et al., 2004).

Considering characters and management flexibilities of the proposed privately-owned social housing model, the private real estate developer generally has two real options at decision making stage, which are a deferral option and an abandonment option separately. As for the abandonment option, it is the last choice and will not be exercised in general conditions because of the irreversible investments, e.g. construction costs. So, the only considered real option is the deferral option (Li et al., 2014). In addition, referring to Article 52 of the "Act for Promotion of Private Participation in Infrastructure Projects", if, during the building or operations of an infrastructure by a private institution, there is any serious schedule delay, material defects in the construction quality, poor operation, or other major events, the authority in charge shall order the private institution to make improvements within a given period with the concession agreement. The authority in charge can suspend part or all of the construction or operations if no improvement is achieved within the given period or if the improvement is ineffective. With reference to Article 3 of the "Regulations For Private Institution Applying to Participate in the Infrastructure Project Under Its Own Planning", where the private institution provides private land needed by the project, it shall, subject to the nature of the project, apply to the authority in charge by submitting the construction plan (including the construction period and the trial operation period, if any) and other documents. Then, the deferral option of a privately-owned social housing project is an American option, which can be exercised at any time without affecting the progress of the construction, since the private real estate developer gets the land use rights.

Multiple models and approaches are available for valuing option premium (OP), whilst Black-Scholes model is most widely used, because of its simplicity in process and accuracy in results. As explained before, deferral option is the principal real option of privately-owned social housing projects at decision making stage, and it should be excised at any time without affecting the progress of the construction, which is so short that the deferral option of privately owned social housing projects can be simplified as European style. As a result, the Black-Scholes model is applicable to privately-owned social housing projects. Furthermore, this basic model is modified by considering the value leakage, which derives from several reasons (e.g. possible construction accompanied cost increase as a result of inflation) in the deferral period.

Like Hui et al. (2011) demonstrated, before constructing the assessment model of option premium of privately-owned social housing projects, three assumptions should be made: ①the value of one privately-owned social housing project follows geometric Brownian motion, and its rate of return is normally distributed; ②the risk-free interest rate and property's price volatility are known and constant throughout the period of development; ③the option is priced in a frictionless market. Then, if the value leakage is  $\delta$ , the value  $S$  at time  $t$  (i.e.  $S_t$ ) of one privately-owned social housing project varies according to a stochastic differential equation in the form of:

$$dS_t = (r_f - \delta)S_t dt + \sigma_s S_t dB_t \tag{2}$$

where  $r_f$  means risk-free interest rate;  $\sigma_s$  is the average volatility of social housing project rent;  $\delta$  means capitalization rate of holding the redeveloped project;  $B_t$  is one-dimensional Brownian motion. Then, based on the risk-neutral assumption and Ito's lemma (Chen, 2007), the stochastic differential equation of the privately-owned social housing project's OP and its boundary condition can be written as:

$$\frac{\partial OP}{\partial t} = r_f OP - (r_f - \delta)S_t \frac{\partial OP}{\partial t} - \frac{1}{2} \sigma_s^2 S_t^2 \frac{\partial^2 OP}{\partial S_t^2} \tag{3-1}$$

$$OP(S_t, T) = \text{Max}[(S_t - C_t), 0] \tag{3-2}$$

where  $C_t$  is the investment cost of the privately-owned social housing project at time  $t$ . Solving the stochastic differential equation of Equation (3-1) & (3-2), it is found that:

$$OP_t = S_t e^{-\delta(T-t)} N(d_1) - C_t e^{-r_f(T-t)} N(d_2) \tag{4-1}$$

$$d_1 = \frac{\left[ \ln\left(\frac{S_t}{C_t}\right) + \left(r_f - \delta + \frac{\sigma_s^2}{2}\right) \right]}{\sigma_s \sqrt{T-t}} \tag{4-2}$$

$$d_2 = \frac{\left[ \ln\left(\frac{S_t}{C_t}\right) - \left(r_f - \delta + \frac{\sigma_s^2}{2}\right) \right]}{\sigma_s \sqrt{T-t}} = d_1 - \sigma_s \sqrt{T-t} \tag{4-3}$$

where  $T$  is the maturity time of the privately-owned social housing project's deferral option;  $N(d_1)$  and  $N(d_2)$  are cumulative probabilities of the variable smaller than  $d_1$  and  $d_2$  separately.

The conventional NPV method has been repeatedly criticized for its disabilities in dealing with



uncertainty, irreversibility and management flexibility, while these disabilities can be cured by ROV. However, as a broadly utilized decision making approach, the NPV method has many undeniable advantages (Chen, 2007). Trigeorgis (2005) put forward a new expanded NPV criterion to capture the additional value of managerial operating flexibility and other strategic interactions:

Expanded (or strategic) NPV (ENPV) = passive NPV + Option Premium (OP) (ROV, Flexibility value and Strategic value).

Based on such a model, it may now be justified to accept projects with negative NPV of expected cash flows (if this is offset by a larger option premium as a result of additional flexibility and strategic value), or delay investment with positive NPV until a later time when expanded NPV would be maximized under uncertainty (Trigeorgis, 2005). This model is adopted, optimized and exemplified in this study to value privately-owned social housing projects.

Since this study aims to value the privately-owned social housing project at decision making stage,  $t$  becomes 0 in Equation (4-1) ~ (4-3) combining Equation (1), the real option-based valuation model for the privately-owned social housing project at decision making stage can be written as:

$$ENPV = \sum_{t=0}^n \frac{(C_t - CO_t)}{(1+i_c)^t} + S_0 e^{-\delta T} N(d_1) - C_0 e^{-r_f T} N(d_2) \quad (5-1)$$

$$d_1 = \frac{\left[ \ln\left(\frac{S_0}{C_0}\right) + \left(r_f - \delta + \frac{\sigma_s^2}{2}\right) \right]}{\sigma_s \sqrt{T}} \quad (5-2)$$

$$d_2 = \frac{\left[ \ln\left(\frac{S_0}{C_0}\right) - \left(r_f - \delta + \frac{\sigma_s^2}{2}\right) \right]}{\sigma_s \sqrt{T}} = d_1 - \sigma \sqrt{T} \quad (5-3)$$

#### 4. Case study

##### 4.1 Basic information

To exemplify above proposed model, this study takes the four social housing cases (three of which have been constructed) currently open for rent in Taichung City as examples. The four cases are not provided by private organizations via the construction method (are not privately-owned social housing projects), of which three are government self-built, and one is a case of donation (divided back) for volume awards. The four cases are all provided by local governments and constructed by the private real estate developer. That is to say, these studied four social housing cases (case A, case B, case C, and case D) are hypothetical privately-owned social housing projects with real datas. Main indices of these projects are extracted from the information published by the Taichung City Government Housing Development Department (2020) and illustrated in Table 2.

Table 2. Main indices of the studied social housing project

CASE A Index name	CASE A Index value	CASE A Index name	CASE A Index value
Land area (10 <sup>4</sup> m <sup>2</sup> )	0.5423	Building area of social housing buildings (10 <sup>4</sup> m <sup>2</sup> )	1.2161
Social housing suites	200	Building area of commercial buildings (10 <sup>4</sup> m <sup>2</sup> )	0.0538
Parking spaces for motor vehicles	73	floors above ground	13
Motorcycle Parking	300	floors underground	2
CASE B Index name	CASE B Index value	CASE B Index name	CASE B Index value
Land area (10 <sup>4</sup> m <sup>2</sup> )	0.5339	Building area of social housing buildings (10 <sup>4</sup> m <sup>2</sup> )	1.2369
Social housing suites	196	Building area of commercial buildings (10 <sup>4</sup> m <sup>2</sup> )	0.0465
Parking spaces for motor vehicles	110	floors above ground	9
Motorcycle Parking	241	floors underground	1
CASE C Index name	CASE C Index value	CASE D Index name	CASE C Index value
Land area (10 <sup>4</sup> m <sup>2</sup> )	0.1704	Building area of social housing buildings (10 <sup>4</sup> m <sup>2</sup> )	1.5141
Social housing suites	190	Building area of commercial buildings (10 <sup>4</sup> m <sup>2</sup> )	0.074
Parking spaces for motor vehicles	74	floors above ground	14
Motorcycle Parking	218	floors underground	3
CASE D Index name	CASE D Index value	CASE D Index name	CASE D Index value
Land area (10 <sup>4</sup> m <sup>2</sup> )	0.8535	Building area of social housing buildings (10 <sup>4</sup> m <sup>2</sup> )	1.9385
Social housing suites	300	Building area of commercial buildings (10 <sup>4</sup> m <sup>2</sup> )	0.2195
Parking spaces for motor vehicles	126	floors above ground	14
Motorcycle Parking	393	floors underground	2

## 4.2 Model parameters

### 4.2.1 NPV-relative parameters

According to the proposed valuation model, a lot of parameters should be determined in advance to calculate the NPV of this social housing project, primarily including  $CIt$ ,  $COt$ , and  $q$ .

The first parameter about  $CIt$  is the average letting rate of social housing buildings and other affiliated businesses (such as commercial buildings). After investigation the supply and demand of the surrounding rental housing market in the four cases, the local demand of social housing buildings and other affiliated businesses (such as commercial buildings) is significantly greater than the supply, the final average letting rate is evaluated at 100%.

The second parameter about CI<sub>t</sub> is the average monthly rent of social housing buildings. In Taichung City, the current situation is set at 50% of the market price in the first year, set at 60% of the market price in the second year, and set at 70% of the market price in the third year. The low-income or middle-income households that meet the requirements of Article 4 of the "Housing Act" will be set at 65% of the market price from the third year. Finally, they average rent income was set at 60% of the market price based on the concept of average calculation at first. According to market research and comparative analysis, the average monthly rent of similar private rental houses nearby is NT\$165 / m<sup>2</sup>, NT\$140 / m<sup>2</sup>, NT\$252 / m<sup>2</sup>, and NT\$242 / m<sup>2</sup>, respectively. After a 40% discount, the average monthly rent of social housing buildings in Cases A, B, C, and D is NT\$99 / m<sup>2</sup>, NT\$84 / m<sup>2</sup>, NT\$151 / m<sup>2</sup>, and NT\$145 / m<sup>2</sup>, respectively. However, considering the operating period of this case is evaluated in 50 years, in the long run, it is relatively reasonable to evaluate the average monthly rent of social housing buildings based on a 30% discount on the market price. After the 30% discount, the average monthly rent of social housing buildings in Cases A, B, C, and D is NT116 / m<sup>2</sup>, NT98 / m<sup>2</sup>, NT176 / m<sup>2</sup>, and NT169 / m<sup>2</sup>, respectively.

The third parameter about CI<sub>t</sub> is the average monthly rent of other affiliated businesses (such as commercial buildings). According to market research and comparative analysis, the average monthly rent of similar private rental houses nearby is NT\$177 / m<sup>2</sup>, NT\$150 / m<sup>2</sup>, NT\$231 / m<sup>2</sup>, and NT\$275 / m<sup>2</sup> respectively. This study uses these data directly. In addition, according to the prediction of the future inflation rate of Taichung City, it is assumed that the rent of social housing buildings and other affiliated businesses (such as commercial buildings) will increase by 9% every 3 years (estimated at 3.00% per year).

The fourth and fifth parameters related to CI<sub>t</sub> are the monthly income of parking spaces for motor vehicles and motorcycle parking. The income is determined by its number, average occupancy rate and monthly rent. According to the market supply and demand situation, the local demand for parking spaces for motor vehicles and motorcycle parking is obviously greater than the supply. Therefore, the final average occupancy rate is evaluated at 100%. According to market research and comparative analysis, it is estimated that the monthly rents for each parking spaces for motor vehicles in Cases A, B, C, and D are is NT\$2,100 / unit, NT\$1,400 / unit, NT\$2,500 / unit, and NT\$2,500 / unit, respectively. It is also estimated that the monthly rent for each motorcycle parking in the four cases is NT\$100 / unit.

Taking Case A as an example here, the calculation process of the average monthly rent of social housing buildings and the average monthly rent of market price is shown in Table 3.

Table 3. Calculation table of the average monthly rent of social housing buildings and market price

Case A social housing building

Room type	Rental area (m <sup>2</sup> )	Number of households	The total area of each room type (m <sup>2</sup> )	Rent after 40% off (Including management fee) (NT\$ / month)	Rental unit price after 40% off (NT\$ /m <sup>2</sup> / month)	Total rental price after 40% discount for each room type (NT\$ / month)
One m bedroo	39.40	36	1,418.40	4,600	117	165,600
One m bedroo	44.33	52	2,305.16	5,000	113	260,000
One m bedroo	39.80	12	477.60	4,600	116	55,200
One m bedroo	39.07	20	781.40	4,400	113	88,000
Two m bedroo	78.64	40	3,145.60	7,000	89	280,000
Two m bedroo	77.95	20	1,559.00	6,900	89	138,000
Three m bedroo	123.67	20	2,473.40	10,800	87	216,000
Subtotal		200	12,160.56			1,202,800
Average rent unit price of all social housing buildings after 40% off (= 1,202,800÷12,160.56) (NT\$ /m <sup>2</sup> / month)			99	Average rent unit price of all social housing buildings calculated base on market price (=99÷0.6) (NT\$ /m <sup>2</sup> / month)		
					165	

Remarks: Considering that the operation period of this case is evaluated in 50 years, in the long run, the average monthly rent of social housing buildings should be more reasonable based on the market price of 30%. After a 30% discount, the average monthly rent of social housing buildings in case A is NT\$116 / m<sup>2</sup> (= NT\$165 / m<sup>2</sup> \* 0.7).

The first parameter about COt is the purchase cost of land. In the case of social housing built by the private organizations via the construction method, considering the financial feasibility, the land will be rented to construct social housing, and the rent is estimated based on the 4.5% of declared value by reference to the provisions in Table 1. The second parameter about COt is the capital expenditures (construction cost of buildings), includes plan, design and construction supervision cost, direct construction cost, project administration fees and others (including

project management cost, price adjustments, the cost of public art, various administrative fees, etc.). After referring to the “Public Construction Project Expenditure Compilation Manual for Construction Projects” formulated by the Public Construction Commission, Executive Yuan, in Taiwan, and the preliminary planning research report on the relevant social housing in Taichung City, the total capital expenditures in Cases A, B, C, and D are NT\$538.95 million, NT\$374.64 million, NT\$61.783 million, and NT\$91.31 million, respectively.

The third parameter about COt is the design and construction supervision cost. According to market research, it can be estimated as NT\$5 million per case. The fourth parameter about COt is loan interest. According to the latest statistics from the Central Bank of the Republic of China (Taiwan), the annual interest rate of borrowed funds is estimated at 2.63%. In addition, the self-owned funds in the investment amount are estimated at 40%, and the borrowed funds are estimated at 60%. These loans will be borrowed from local commercial banks equally in three years of construction stage, and then repaid with maximum capacity in the operation stage.

The fifth parameter about COt is the total loan interest. It is estimated that the total loan interest in case A, case B, case C, and case D is NT\$279.73 million, NT\$19.445 million, NT\$ 32.067 million, and NT\$467.8 million, respectively. The sixth parameter about COt is the operating expenditures, including management and maintenance expenses costs, repair and maintenance costs, fire and earthquake insurance premiums, and depreciation costs. After referring to the preliminary planning research report of the relevant social housing in Taichung City, it is estimated that the operating expenditures of Case A, Case B, Case C, and Case D during the 50-year operation period is NT\$961.7 million, NT\$66.851 million, NT\$112.45 million and NT\$ 1.683 billion. The seventh parameter and the eighth parameter related to COt are the depreciation period of social housing buildings and affiliated commercial buildings. After referring to the relevant laws and regulations, both parameters are assumed to be 50 years.

As for  $q$ , refer to the preliminary planning research report of relevant social housing in Taichung City, it is evaluated at 3.26%. In addition, referring to Article 22 of the “Housing Act”, during the operation of social housing, rental income from spaces used to provide housing, long-term care services, services for the disabled, child-care services, and nursery, shall be exempt from sales tax.

#### **4.2.2 OP-relative parameters**

In order to calculate OP part in Equation (5-1) ~ (5-3), six parameters are necessary in total. The first parameter is the maturity time  $T$ . According to the aforementioned private real estate developers, after obtaining the land use rights, they can exercise the delayed option of social housing at any time without affecting the construction progress (is an American option). In this case, with reference to the surrounding real estate price changes, the maturity time is estimated as 1 year. That is to say, one year deferral option is considered in valuing this studied social housing project.

The second parameter is the current value  $S_0$ , which is the discounted value at decision making stage and can be estimated on the basis of  $q$  (i.e. 3.26%) and income cash flow at operation stage (i.e. 50 years). Take case A as an example, considering parameters in previous section, the

annual rent income of social buildings and affiliated commercial buildings in first year is NT\$ 19.05 million, which will increase to NT\$19.63 million in second year. With such an increasing trend of rent income, and then deducting operating costs or expenses and profit-seeking enterprise income tax by,  $S_0$  can be calculated as NT\$ 496.84 million.

The third necessary parameter is the investment cost  $C_0$ , which is also a discounted value at decision making stage and can be estimated on the basis of  $q$  and the irreversible investment. The cost for land use rights is paid by the private real estate developer for obtaining the option, and thus it should not be included in the irreversible investment (Chen, 2007). The costs that occur in operation stage should not be included, too (Chen, 2007). Because the operation cost would only occur when the commercial buildings are rented out, and it is not irreversible. The investment cost should only include the irreversible investment part. Based on the total project cost of case A and  $q$  (i.e. 3.26%),  $C_0$  can be calculated as NT \$ 497.98 million.

The fourth necessary parameter is  $\delta$ . From the perspective of data availability, only  $\delta$  induced by cost increase is considered. Take case A as an example, it is obvious that the direct construction cost is the main component of building cost, approximately 91.32% of the construction cost.

In addition, according to the survey, housing construction cost is the main component of the direct construction cost, accounting for about 53% of direct engineering cost. Therefore, the growth rate of total cost can be represented according to the average growth rate of material cost and labor cost, and their proportions are about 60%, 30% respectively. The study selected the local consumer price index (CPI) and per capita salary as their tokens respectively. According to the "National Statistics, R.O.C. (Taiwan)", the annual increases of CPI and per capita salary from 2014 to 2019 are 0.80% and 0.89% separately. Therefore, the value leakage  $\delta$  of the studied social housing project is about 0.758% ( $=60\% * 0.80\% + 30\% * 0.89\%$ ).

The fifth parameter is the risk-free interest rate  $r_f$ , which is usually based on the yield of the government's debt (Chen, 2007). On November 08, 2019, the Central Bank of the Republic of China (Taiwan) announced to sell a thirty-year period national debt with an annual interest of 0.95%, which is adopted as the risk-free interest rate. That is to say,  $r_f$  of the studied social housing project is 0.95%.

The sixth parameter is the average volatility of social housing rent. Because social housing is a new type of affordable housing in Taiwan, there is no special statistical data on social housing rent. But, as mentioned before, social housing rent is usually 70% of the rents of similar private rented houses nearby, which are positively correlative to their selling prices. So, the average volatility of commodity houses' selling price can represent that of social housing rent. From the official real estate information platform in Taiwan, from the third quarter of 2014 to the third quarter of 2019, the average fluctuation of the Taichung City residential price index was 3.68%, as shown in Table 4.

Table 4. Taichung City Residential Price Index and its volatility from the 3rd quarter of 2014 to the 3rd quarter of 2019

Point in time	Residential price index	Residential price index rate of change to the same quarter last
Q3 2014	95.5	12.82%
Q3 2015	100.63	5.37%
Q3 2016	101.65	1.01%
Q3 2017	101.81	0.16%
Q3 2018	102.43	0.61%
Q3 2019	104.59	2.11%
Average value		3.68%

In Table 5, a summary of the relative parameters and values of NPV in case A of this study is listed.

Table 5. a summary of the relative parameters and values of NPV in case A

Variable	Symbol	Description	Value
NPV-relative parameters	q	Discount rate	3.26%
	CI <sub>t</sub>	Cash inflow of the privately-owned SH project at year t	Shown below
Determination of CI <sub>t</sub>	CO <sub>t</sub>	Cash outflow of the privately-owned SH project at year t	Shown below
		Maximum operation period for SH buildings	50 years
		Maximum operation period for possible affiliated commercial buildings	50 years
		The average letting rate of SH buildings and affiliated commercial buildings	100%
		The average rent of SH buildings	165 NTD/m <sup>2</sup>
		The average rent of affiliated commercial buildings	248 NTD/m <sup>2</sup>
		Increase rate of the rent of SH buildings and affiliated commercial buildings	3%
		The monthly rent of a parking space	2,100 NTD
		The monthly rent of a motorcycle parking space	100 NTD
	Determination of CO <sub>t</sub>		Land cost(Rent of soil) in first year
		The preliminary planning and research expenses	5 million NTD
		Capital expenditure (building cost)	538.95 million NTD
		The annual loan interest rate	2.63%
		The total loan interest	279.73 million NTD
		The depreciation periods of SH buildings	50 years
		The depreciation periods of affiliated commercial buildings	50 years

5. Results and discussions

Putting above NPV-relative parameters into Equation (1), the NPV in case A, case B, case C, and case D can be calculated as NT\$-1.14 million, NT\$-18.63 million, NT\$-9.68 million, and NT\$-18.49 million, respectively. Since each NPV is negative, these studied social housing projects seem financially unacceptable, and thus the private real estate developer should not undertake such a privately-owned social housing project. Then, putting above OP relative parameters into Equation (5-2) and (5-3),  $d_1$  and  $d_2$  can be calculated as 0.07 and -0.08 separately in case A,  $d_1$  and  $d_2$  can be calculated as 0.02 and -0.13 separately in case B,  $d_1$  and  $d_2$  can be calculated as 0.06 and -0.09 separately in case C, and  $d_1$  and  $d_2$  can be calculated as 0.05 and -0.10 separately in case D. Further,  $N(d_1)$  and  $N(d_2)$  can be dug out from the value table of the standard normal distribution function as 0.5199 and 0.4602 respectively in case A,  $N(d_1)$  and  $N(d_2)$  can be dug out as 0.5080 and 0.4483 respectively in case B,  $N(d_1)$  and  $N(d_2)$  can be dug out as 0.5239 and 0.4641 respectively in case C, and  $N(d_1)$  and  $N(d_2)$  can be dug out as 0.5199 and 0.4602 respectively in case D.

Then, putting  $N(d_1)$  and  $N(d_2)$  in each case, as well as other parameters obtained in section OP-relative parameters into Equation (5-1), the OP values of Case A, Case B, Case C, and Case D can be calculated as NT\$29.42 million, NT\$11.43 million, NT\$29.37 million, and NT\$40.56 million, respectively. Then, adding OP generated by one-year deferral option and passive NPV together, the ENPV of this studied privately-owned social housing projects in Case A, Case B, Case C, and Case D can be calculated to be NT\$28.28 million, NT\$-7.2 million, NT\$19.69 million, and NT\$22.07 million. In Table 6, a summary of the relative parameters and values of OP in case A of this study is listed.

Table 6. a summary of the relative parameters and values of OP in case A

Variable	Symbol	Description	Value
OP-relative parameters	T	The maturity tim	1 year
	$S_0$	The current value	NT\$496.84 million
	$C_0$	The investment cost	NT\$497.98 million
	$\delta$	The value leakage of the PRH project	0.75%
	$r_f$	The risk-free interest rate	0.95%
	$\sigma_s$	The average volatility of SH rent	3.68%
	$d_1$	$\frac{[\ln(S_0/C_0) + (r_f - \delta + \sigma^2/2)]}{\sigma\sqrt{T}}$	
$d_2$	$\frac{[\ln(S_0/C_0) - (r_f - \delta + \sigma^2/2)]}{\sigma\sqrt{T}}$		-0.08
$N(d_1)$		The cumulative probabilities of the variable smaller than $d_1$	0.5279
$N(d_2)$		The cumulative probabilities of the variable smaller than $d_2$	0.4681
OP		The option premium, $S_0 e^{-\delta T} N(d_1) - C_0 e^{-r_f T} N(d_2)$	NT\$29.42 million
ENPV		Real option valuation model at decision making stage of the privately-owned social housing projects	NT\$28.28 million



Consequently, the studied social housing projects in Case A, Case B, Case C, and Case D become financially acceptable, if taking OP generated by one-year deferral option into consideration. Although case B is still not financially acceptable (case B is relatively poor in location), after considering one-year deferral option, its ENPV will be NT\$29.37 million more than its NPV. In other words, private real estate developer should build, own and operate (BOO) these four social housing projects, and waiting one year is a better choice than building it immediately.

Regarding these cases of this study, there are five potential influencing factors that will affect ENPV. As for income part, the income from social housing buildings is the biggest contributor, and thus the average rent of social housing buildings is a potential factor. As for the cost part, most cost items (except financial expenses) cannot be changed. Another potential factor is the building area of affiliated commercial buildings, whilst it will affect the cost and income part simultaneously. Among OP-relative parameters,  $S_0$  and  $C_0$  are both affected by above-mentioned NPV relative parameters, while  $\delta$ ,  $r_f$  and  $S$  are all beyond private real estate developers' control. Among the OP-relative parameters,  $T$  is also a potential factor. After private real estate developers obtaining the land use rights, they can exercise the delayed option of social housing at any time without affecting the construction progress. In addition, the rate of return on self-owned funds and interest rate of loan funds are also potential factors.

The influences of these five potential influencing factors on the ENPV of this studied social housing project are calculated and demonstrated in Fig. 5 to Fig. 8. Taking Figure 5 as an example, through sensitivity analysis, it is found that the average rent increase rate of social housing is the most influential indicator affecting the fluctuation of the ENPV. When the average rent drops by less than 3%, the ENPV will become negative. The second largest influencing indicator is the interest rate of loan funds. Taking case A as an example, when it increases by about 5%, the ENPV will become negative. As regards the maturity time and the building area of affiliated commercial buildings, their influences are too small to be negligible.

## 6. Conclusions

In Taiwan, Since 2011, the government has promoted a policy of renting and not selling social housing, and through related plans, it has assisted socio-economically disadvantaged households to rent housing. The competent authorities have announced many incentive policies to promote the private sector's provision of social housing. Based on the limited financial resources of the government, a privately-owned provision mode of social housing projects is created in this study. Then, utilizing ROT, ROV for this new model is proposed, followed by a case study of the four social housing cases (three of which have been completed for construction) currently open for rent in Taichung City to conduct relevant research.

This study proposes a new social housing provision mode (i.e. privately-owned social housing) through employing BOO financing scheme. Based on related management flexibilities of this mode at decision making stage, only the deferral option is considered in this study. Then, a real option-based valuation model for privately-owned social housing projects is constructed.

The proposed social housing provision mode and relevant valuation model are exemplified in the four social housing projects in Taichung city, which seems financially unacceptable because of a negative NPV indicator. Then, adding OP generated by one-year deferral option and passive NPV together, the ENPV indicator becomes positive in three of these cases and thus these three social housing projects turn to financially acceptable. Although the deferral option is merely considered in the proposed valuation model, the OP is big enough to offset the negative NPV of studied social housing projects. Therefore, under the proposed social housing provision model, the private sector will have more management flexibilities and corresponding real options in the life cycle of a social housing project. As a result, the proposed social housing provision mode is a feasible way to attract the private sector.

Finally, through sensitivity analysis, it is found that the average rent increase rate of social housing is the most influential indicator affecting the fluctuation of the ENPV. Moreover, with rich experiences in commercial housing projects and multiple options in public rental housing projects, the private developers may adapt to and even benefit from some uncertainties. As a result, to better assess the value of PRH projects in BOOC mode, the real option theory and its pricing model are introduced (Li et al., 2016). The valuation model needs to be intensively studied and applied in more projects. Besides, identifying other real options (such as option to switch) of a privately-owned social housing project will be future works of this study.

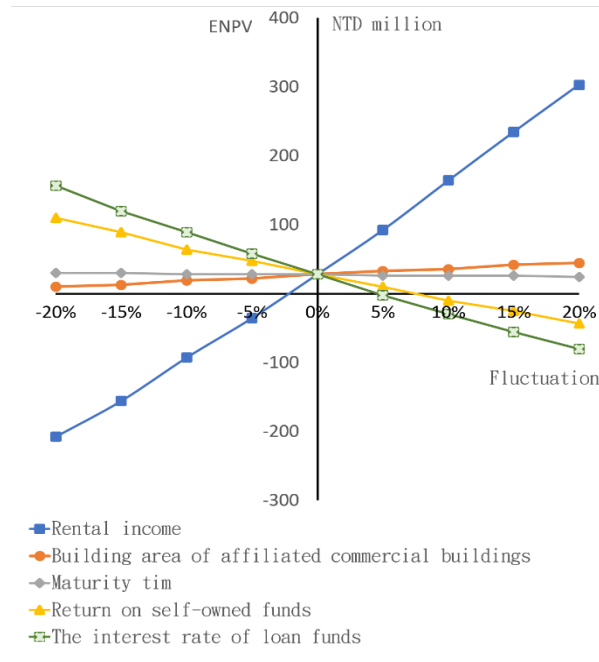


Fig. 5. Results of the sensitivity analysis in case A

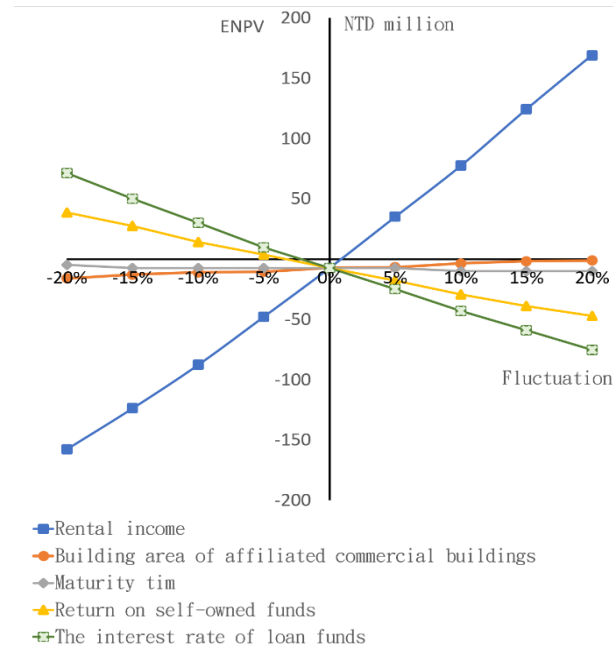


Fig. 6. Results of the sensitivity analysis in case B

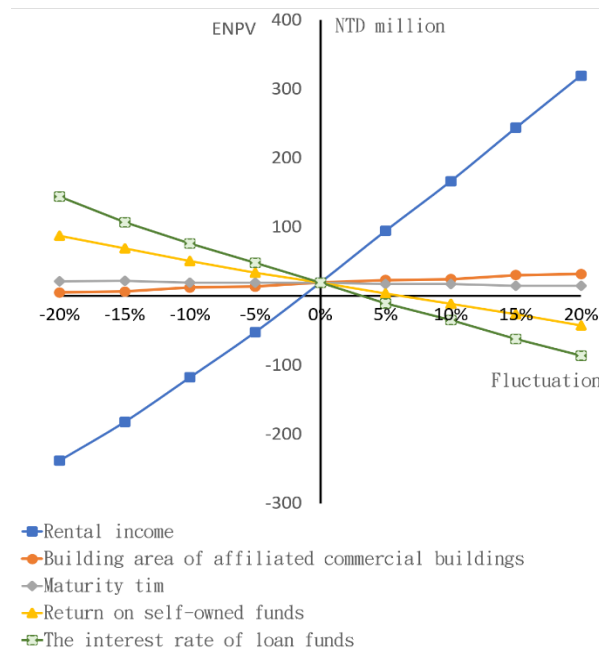


Fig. 7. Results of the sensitivity analysis in case C

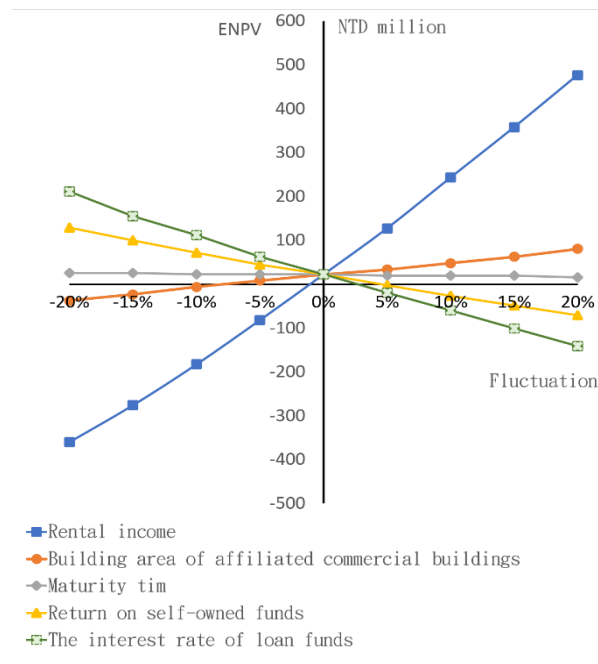


Fig. 8. Results of the sensitivity analysis in case D

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